AN-107
APPLICATION NOTE

Active Feedback Improves Amplifier Phase Accuracy

by James Wong

Using matched op amps and active feedback, you can minimize the phase error and so extend the bandwidth of an amplifier by more than an order of magnitude. This technique is cheaper than using wideband amplifiers and less sensitive to the temperature-related drift specifications of passive components.

In applications such as sonar and image-processing systems, the phase relationship between two or more signals reveals essential information. These systems require accurate phase response in their amplifier circuitry to minimize measurement errors. In such cases, active feedback can often serve much better than other approaches. A typical op amp is insufficient in this situation because it introduces significant phase shift long before it reaches its −3dB frequency. The consequent phase error reduces the effective bandwidth of an op amp to something significantly less than the −3dB point.

You can use a wideband amplifier to overcome this phase-error problem. If the wideband amplifier operates with a −3dB bandwidth that is much higher than that of the signal that you intend to amplify, then the phase error at your signal's frequency decreases proportionately. The wideband amplifier's greater expense is the main drawback to this approach.

If you cascade two or three amplifiers, each of which has its gain reduced to share the overall gain of the composite amplifier, the gain reduction at each stage of the amplifier increases the −3dB bandwidth of each stage of the amplifier. Consequently, the overall bandwidth for a given phase accuracy is increased, but you pay for this improvement with increasingly higher costs and noise levels.

Another, less expensive way to solve the phase-error problem is to introduce extra circuitry in the amplifier's feedback loop, which provides frequency compensation. You can use an RC circuit to create a zero in the feedback loop that cancels the amplifier's pole. This cancellation improves the phase response markedly by lessening the amplifier's phase-response roll-off. The chief disadvantage of the RC technique is that it requires extensive tuning to match the zero with the pole. Furthermore, the different temperature coefficients of the RC components cause the zero to drift. And when the zero drifts, it no longer cancels the pole, and phase error becomes a problem once again.

Placing an op-amp circuit in the feedback stage of the amplifier creates the active feedback that can overcome the temperature drift of the RC networks. It is also a thriftier approach than using a wideband amplifier. You must make sure, however, that the op amps are very closely matched.

Monolithically matched dual or quad op amps can provide the frequency-matching characteristics (to within 1 to 2%) necessary for the success of the active-feedback approach. This close matching is necessary across the full temperature range of your application. It's easier to achieve in an integrated dual or quad op amp than it is in discrete resistors and capacitors.

Figure 1 shows a basic active-feedback circuit. It requires an op amp and two external resistors to achieve phase-error cancellation. In the circuit, op amp A provides the forward gain of the composite amplifier. Resistors R_1 and R_1/K_1 determine the closed loop gain $A_V \gg 1 + K_1$. Amplifier A_2 provides active feedback to op amp A_1. The ratio of resistors R_2 and R_2/K_2 determines the amount of phase-error compensation and has no effect on the forward gain of the composite amplifier. You obtain optimum error cancellation when $K_1 = K_2$.

In terms of the complex frequency response, the error terms for the circuit are given by:

$$\text{magnitude error} \cong \left(\frac{\omega}{\beta\omega_T}\right)^2$$

$$\text{phase error} \cong -\left(\frac{\omega}{\beta\omega_T}\right)^3.$$

These equations let you compare the phase error associated with a single amplifier with that associated with the 2-op-amp, active-feedback approach. They arise from a complete analysis of the basic circuit and its derivatives (see "Analyzing compensation techniques").
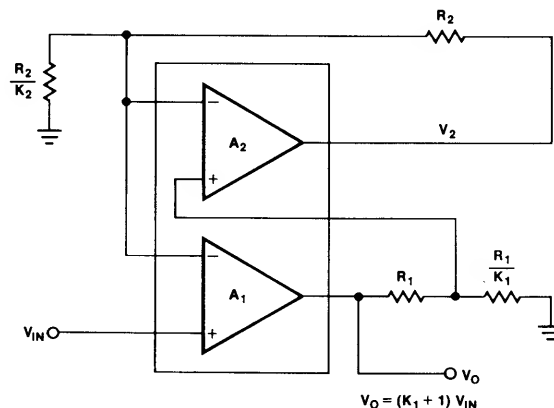

$$V_O = (K_1 + 1) V_{IN}$$

FIGURE 1: *Second-order compensation, as provided by the op amp in the feedback path of this circuit, extends the effective bandwidth of an amplifier substantially.*

TABLE 1: AC-Error Comparison

FREQUENCY	SINGLE-STAGE CONVENTIONAL PHASE (DEGREE)	CASCADED (TWO-STAGE) PHASE (DEGREE)	SECOND-ORDER COMPENSATION PHASE (DEGREE)
5kHz	−0.57	−0.36	0
10kHz	−1.15	−0.72	−0.0005
50kHz	−5.7	−3.62	−0.06
100kHz	−11.3	−7.21	−0.46
500kHz	−45.0	−45.0	−45.0

FIGURE 2: *Phase shift for single-op-amp, cascaded-op-amp, and second-order-compensation designs are charted here. Note the significant improvement brought about by second-order compensation.*
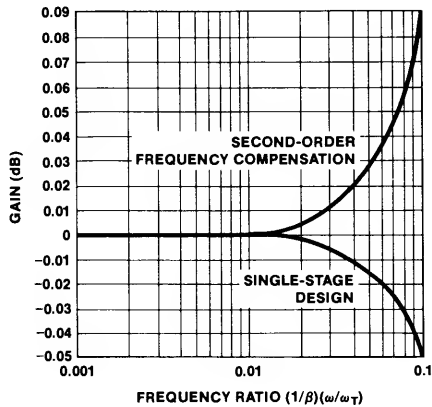

FIGURE 3: *Second-order compensation does carry the penalty of gain peaking, but this doesn't become significant until close to the corner frequency.*

DOMINANT POLES OCCUR AT 500kHz

Table 1 tabulates the phase error and magnitude error for an amplifier with a gain of 10. The comparison assumes that the op amp's unity-gain bandwidth is 5MHz. As you can see in the table, the dominant poles of all three amplifiers occur at 500kHz, where the phase shift for each technique is −45°. Clearly, the compensation techniques do not extend the bandwidth of the op amp itself. Rather, these second-order compensation techniques reduce the phase error at lower frequencies by adding an equal but opposite phase shift in the amplifier feedback loop.

If you must limit phase error to less than half a degree, the second-order compensation technique increases the effective bandwidth of your amplifier from about 5kHz to more than 100kHz. If you need to limit phase error entirely, the second-order compensation reduces phase error virtually to zero at frequencies to 50kHz; in contrast, a single-stage amplifier would be limited to well below 500Hz.

The phase and magnitude responses are plotted in Figures 2 and 3, respectively. The single op-amp response serves as a basis for comparison. The maximum scale of 1.0 represents the −45° phase-shift frequency. Figure 2 shows that the cascaded 2-stage amplifier offers only a slight improvement in bandwidth, and that the second-order compensation method offers a significant improvement in effective bandwidth.

THE TRADEOFFS ARE MINOR

The tradeoffs associated with the second-order feedback technique are minimal. Figure 3 illustrates the appreciable gain peaking incurred, but the circuit typically peaks about 3dB at the −45° phase frequency, well outside the useful range. Within the frequency range where phase shift is negligible, the gain error is also insignificant. In Figure 3, for example, at 1/10 of the corner frequency, gain error is only 0.1dB, about 1.2%.

In executing the second-order compensation design, it's extremely important to use op amps with frequency responses matched to within 1 to 2%. Op amps packaged separately can have mismatches as high as 10 to 20%, and high levels of mismatching cause either over- or undercompensation. Overcompensation creates excessive phase peaking, and undercompensation causes early phase roll-off.

Theoretically, second-order compensation works for any closed-loop gain. In practice though, at low gains, within the 1-to-5 range, the circuit may become unstable due to phase-margin degradation introduced by the active feedback. As a rule of thumb, you should work with a gain of 10 or greater. The match between theoretical and actual performance improves as the gain of the circuit is increased.

A circuit using the second-order compensation is illustrated in Figure 4. The op amp used in this example is an OP-470, which is unity-gain stable and provides a 6MHz unity-gain bandwidth. It's a quad op amp that offers a 1% match in ac characteristics between the four op amps on the chip. The circuit provides a gain of 10 for the amplifier.

The actual phase response of the circuit was measured using a network analyzer and is compared with a single-stage amplifier in Figure 5. The measurement confirms that the phase shifts of the second-order-compensated design and the single-stage design converge at −45° and −135°, respectively. The second-order response runs virtually flat with negligible phase shift to a much higher frequency before bandwidth limitation sets in. The roll-off is much steeper for the second-order system than for the single-stage system. Figure 6 illustrates more clearly that the second-order circuit's phase error remains nearly zero out to 100kHz, while the single-stage amplifier's low phase-shift bandwidth is limited to 2kHz.

Figure 6 also reveals a slight amount of phase peaking at about 100kHz, just before the response rolls off. The peaking stems from the op amps' second poles near the unity-gain frequency. With a closed-loop gain of 10, the dominant pole and the second pole are separated by only one decade in frequency. This proximity causes secondary effects on the phase response of the amplifier. However, if you increase the closed-loop gain of the circuit, you further separate the two poles, reducing the second-pole effect.

Fortunately, the second-order compensation's improvement in phase error does not exact much of a penalty in magnitude error. Figure 7 charts the magnitude response versus frequency. Some gain peaking is apparent, but as predicted, the peaking occurs well beyond the amplifier's useful bandwidth.
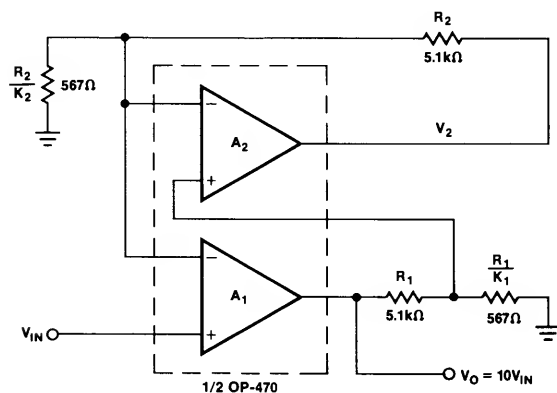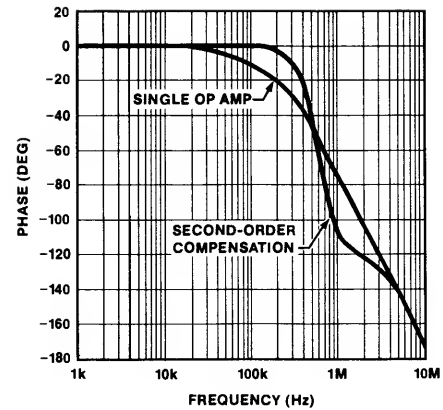

FIGURE 5: *The phase response of Figure 4's circuit illustrates the improved flatness of response that can be attributed to second-order compensation.*
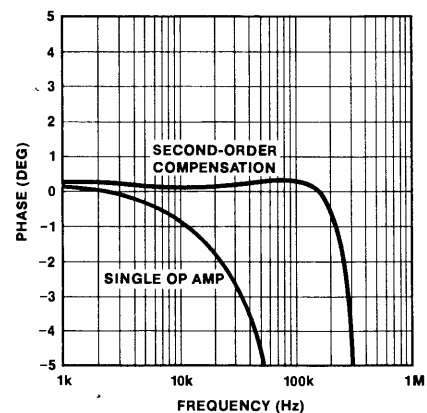

FIGURE 6: *Slight peaking in the phase response at 100kHz for the second-order-compensated circuit is illustrated in this close-up view of Figure 5.*

FIGURE 4: *This amplifier uses the second-order compensation technique with circuit values shown for a gain of 10.*
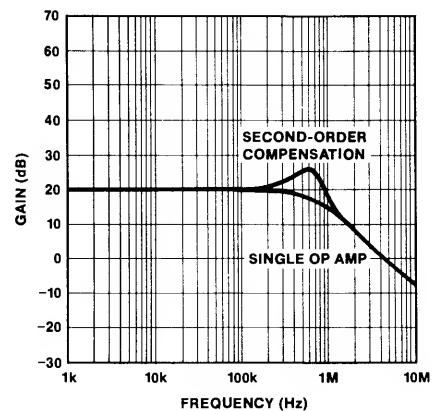

FIGURE 7: *The magnitude response of the second-order-compensated circuit exhibits peaking at a frequency that is well beyond the useful range of the circuit.*

(Figure 8 shows a close-up of the gain peaking for the second-order-compensated circuit.) In the frequency range below 100kHz, gain error is held to less than 0.25dB or 3%. In comparison to the single-stage approach, a second-order compensation offers both low phase error and low magnitude error over a significantly expanded bandwidth.

Either resistor tolerances or op-amp mismatches can degrade the compensation circuit's performance but to no great extent. What does make a big difference is variations in K_2 (Figure 9). If you assume that the op amps in Figure 1 match perfectly, the resistor ratio, K_2, is the only variable that affects the amount of phase compensation that the feedback circuit provides. If you decrease K_2, the gain of op amp A_2 is also decreased; in turn, the dominant pole of A_2 shifts to a higher frequency, thereby decreasing the compensation. In the extreme case, where op amp A_2's gain is reduced to unity, the circuit behaves as if it had no compensation at all. It then responds as a single-stage amplifier. On the other hand, if you increase K_2, the gain of A_2 is likewise increased, and A_2's bandwidth decreases; overcompensation results. The end effect is that phase and gain peak more.

Third-order compensation, which requires another op amp and two resistors in the feedback path, provides even greater phase compensation than afforded by second-order compensation. The schematic for the third-order compensation circuit is given in Figure 10. The additional compensation comes from op amp A_3 in the feedback path of op amp A_2. The basic circuit is the same as that of the second-order compensation circuit.

The sinusoidal transfer function when the phase difference is minimized may be written:

$$\frac{V_O}{V_{IN}} = \left(1 + K_1\right)\left[\frac{1 - j(\tau\omega)^5}{1 - 0.618034(\tau\omega)^2 - 0.618034(\tau\omega)^4 - (\tau\omega)^6}\right]$$

where: $\tau = \dfrac{1}{\beta\omega_T}$.

The numerator contains the phase information and the denominator contains the magnitude information. Table 2 contrasts the magnitude and phase errors for the second-order response with that of the third-order response. Magnitude error is 38% lower for third-order compensation. Phase error is reduced from a third-order to a fifth-order term.

The third-order frequency response is plotted in Figure 11 in comparison to that of the second-order compensation and of the single op-amp design. Given a prescribed error band — for example, one degree of phase error — third-order compensation doubles the effective bandwidth of an amplifier design over the second-order compensation technique.

As you might imagine, the extra op amp in the feedback loop causes the gain to peak more at the $-45°$ corner frequency. However, in the frequency range where phase error is minimal, gain accuracy is also improved. At a frequency ratio below 0.1, magnitude errors do not exceed 0.1dB.
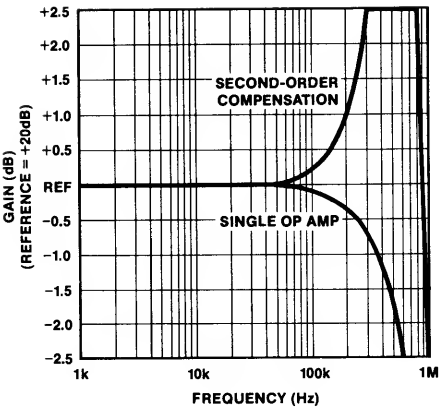

FIGURE 8: *Taking a closer look at the magnitude response, you see that below 100kHz, the frequency range where phase error is negligible, only a small degradation in gain accuracy occurs.*
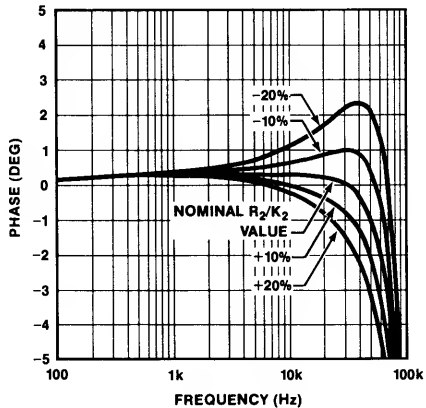

FIGURE 9: *Phase-response sensitivity to variations in the compensation resistor ratio results in slight over- and undercompensation.*

TABLE 2: Error Comparison of Second-Order vs Third-Order Compensation

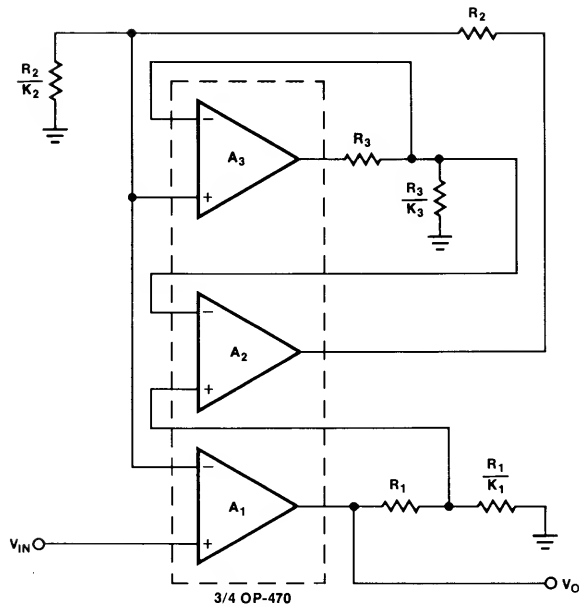|	SECOND-ORDER	THIRD-ORDER
CLOSED-LOOP GAIN	$\dfrac{1}{\beta}$	$\dfrac{1}{\beta}$
MAGNITUDE ERROR	$\left(\dfrac{\omega}{\beta\omega_T}\right)^2$	$0.618\left(\dfrac{\omega}{\beta\omega_T}\right)^2$
PHASE ERROR	$-\left(\dfrac{\omega}{\beta\omega_T}\right)^3$	$-\left(\dfrac{\omega}{\beta\omega_T}\right)^5$

FIGURE 10: *Third-order compensation takes the concept of active feedback one step further, adding another level of compensation to the feedback path.*
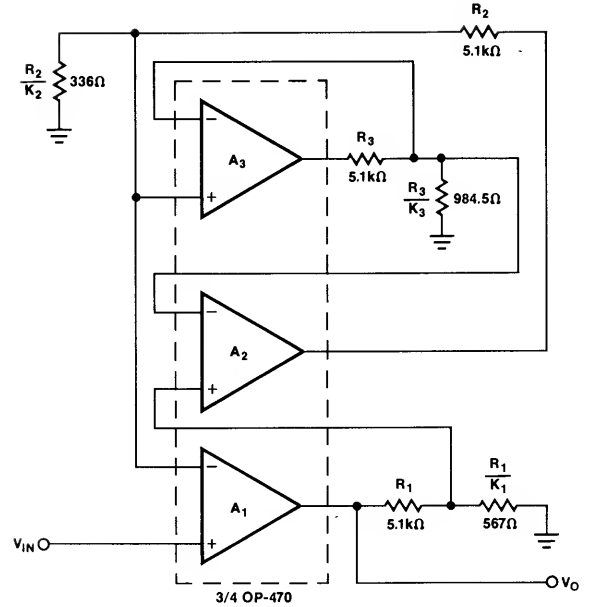

FIGURE 12: *A third-order compensation circuit with component values that set a gain of 10 for the amplifier is illustrated here.*
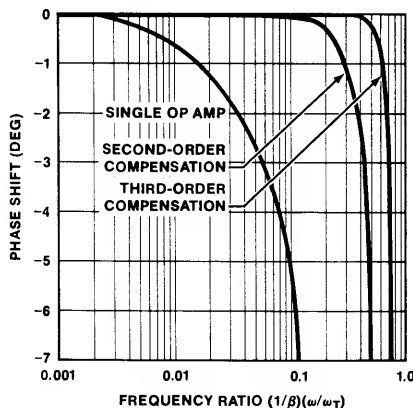

FIGURE 11: *For a given phase error, the third-order compensation technique offers twice the bandwidth that the second-order technique does.*
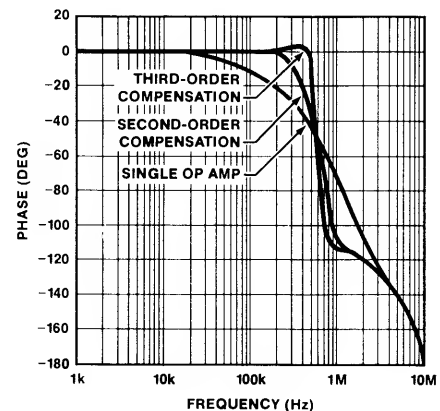

FIGURE 13: *The actual phase response of the third-order circuit remains flat to a higher frequency than the second-order circuit or the single-op-amp design.*

Figure 12 shows a test circuit that implements the third-order compensation. The phase response, measured by a network analyzer, is compared to that of the second-order compensation and of a single-op-amp design in Figure 13. The phase response of the third-order design remains flat beyond 400kHz, double the 200kHz provided by the second-order design. The magnitude response, on the other hand, has considerably higher peaking than the second-order design, but below 350kHz, the third-order design actually provides lower magnitude error than does the second-order design.

REFERENCES

1. Soliman, Ahmed M, "Design of High-Frequency Amplifiers," *IEEE Circuits and Systems,* June 1983.
2. Soliman, AM and Ismail, M, "Active Compensation of Op Amps," *IEEE Transactions on Circuits and Systems,* February 1979.

ACKNOWLEDGMENT

ANALYZING COMPENSATION TECHNIQUES

A single-stage amplifier without any band-limiting circuit, as shown in Figure A, rolls off according to the closed-loop gain of the circuit and the amplifier's own natural frequency response. You can approximate the amplifier's open-loop frequency response in this way:

$$\text{open-loop gain } A(s) \cong \frac{A_O}{1 + \frac{A_O}{\omega_T}s},$$

where A_O is the dc open-loop gain and, ω_T is the unity-gain radian frequency of the amplifier.

The fundamental assumption underlying this approximation is that the amplifier has a single-pole response. Thus, the closed-loop response as a function of frequency is given by:

$$\text{closed-loop gain } A_{CL}(s) = \frac{1}{\beta}\left(\frac{1}{1 + \frac{1}{A_O\beta} + \frac{1}{\beta\omega_T}s}\right).$$

As long as the loop gain is much greater than unity – which is usually the case – you can rewrite the expression as:

$$\text{closed-loop gain } A_{CL}(s) \cong \frac{1}{\beta}\left(\frac{1}{1 + \frac{1}{\beta\omega_T}s}\right).$$

Figure B illustrates the magnitude and phase response of the closed-loop amplifier. Notice that, at the -3dB corner frequency ω_c, the phase shift is -45°. Even at 1/10 of the corner frequency, the amplifier still has a 5.7° phase lag. The maximum frequency must be nearly two decades below the corner frequency to reduce phase error to less than one degree.

Figure 1 illustrates the second-order, active-feedback compensation circuit that you can model by writing the transfer equation for the two amplifier loops. In solving this equation, assume that each amplifier has a single-pole roll-off response and that the two amplifier responses match perfectly.

At the junction of R_1 and R_1/K_1, the resistor divider expression becomes:

$$V_O\left(\frac{1}{1 + K_1}\right).$$

Similarly, the resistor divider expression for R_2 and R_2/K_2 is:

$$V_2\left(\frac{1}{1 + K_2}\right).$$

For ac response, the loop equations can now be written:

$$A_1\text{loop: } V_O = \left[V_{IN} - \left(\frac{1}{1 + K_2}\right)V_2\right]\frac{\omega_T}{s}$$

$$A_2\text{loop: } V_2 = \left[\left(\frac{1}{1 + K_1}\right)V_O - \left(\frac{1}{1 + K_2}\right)V_2\right]\frac{\omega_T}{s}.$$

Solving the simultaneous equations in terms of V_O and V_{IN}, the transfer function is:

$$\frac{V_O}{V_{IN}} = (1 + K_1)\left[\frac{1 + \frac{(1 + K_2)}{\omega_T}s}{1 + \frac{(1 + K_1)}{\omega_T}s + \frac{(1 + K_1)(1 + K_2)}{\omega_T^2}s^2}\right].$$

Defining the time constants:

$$\tau_1 = \frac{1 + K_1}{\omega_T} \text{ and } \tau_2 = \frac{1 + K_2}{\omega_T},$$

and substituting those time constants into the previous equations yields:

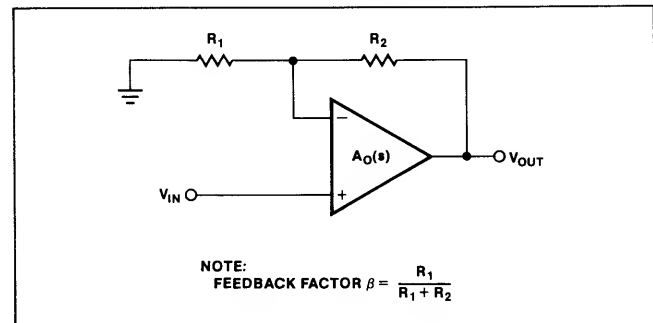$$\frac{V_O}{V_{IN}} = (1 + K_1)\left[\frac{1 + \tau_2 s}{1 + \tau_1 s + \tau_1\tau_2 s^2}\right].$$

NOTE:
FEEDBACK FACTOR $\beta = \frac{R_1}{R_1 + R_2}$

FIGURE A: *A basic single-stage amplifier without any band-limiting circuit rolls off according to the closed-loop gain of the circuit and the amplifier's own natural frequency response.*
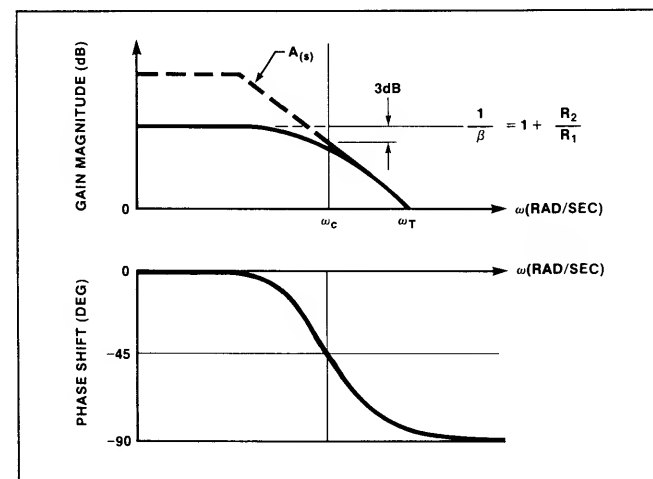

FIGURE B: *Magnitude and phase response are shown for the basic single-stage amplifier. Notice that, at the −3dB corner frequency ω_c the phase shift is −45°.*

ANALYZING COMPENSATION TECHNIQUES *Continued*

For optimum compensation, the time constants τ_1 and τ_2 are made equal:

$$\frac{V_O}{V_{IN}} = \left(1 + K_1\right)\left[\frac{1 + \tau s}{1 + \tau s + \tau^2 s^2}\right].$$

For sinusoidal input, where phase information is important, the complex frequency domain is used:

$$\frac{V_O}{V_{IN}}(j\omega) = \left(1 + K_1\right)\left[\frac{1 + j\tau\omega}{1 + j\tau\omega - \tau^2\omega^2}\right].$$

Now we calculate the complex conjugates to simplify:

$$\frac{V_O}{V_{IN}} = \left(1 + K_1\right)\left[\frac{(1 + j\tau\omega)\left(1 + \tau^2\omega^2 - j\tau\omega\right)}{\left(1 - \tau^2\omega^2\right)^2 + \tau^2\omega^2}\right]$$

$$= \left(1 + K_1\right)\left[\frac{1 - j\tau^3\omega^3}{1 - \tau^2\omega^2 + \tau^4\omega^4}\right].$$

From this equation, the magnitude error expression can be derived.

$$\text{Magnitude error} = \left|\frac{1 - j\tau^3\omega^3}{1 - \tau^2\omega^2 + \tau^4\omega^4}\right| - 1$$

for $\tau\omega \ll 1$,

$$\cong \left|\frac{1}{1 - \tau^2\omega^2}\right| - 1$$

$$\cong 1 + \tau^2\omega^2 - 1$$

$$\cong \tau^2\omega^2.$$

Similarly, the phase error expression can be derived.

$$\text{Phase error} = \text{ARC}\left(1 - j\tau^3\omega^3\right) \cong -\tau^3\omega^3.$$

$\tau\omega$ relates to closed-loop gain as $\dfrac{1}{\beta} = 1 + K_1 = 1 + K_2$,

therefore, $\tau\omega = \dfrac{\omega}{\beta\omega_T}$.

The transfer function can then be rewritten as:

$$\frac{V_O}{V_{IN}} = \frac{1}{\beta}\left[\frac{1 - j\left(\frac{\omega}{\beta\omega_T}\right)^3}{1 - \left(\frac{\omega}{\beta\omega_T}\right)^2 + \left(\frac{\omega}{\beta\omega_T}\right)^4}\right].$$

Using this equation, the phase and magnitude behaviors as functions of frequency are tabulated in Table A. Note that, at 1/10 of the corner frequency, the compensated circuit produces a -0.057° phase shift, far superior to the -5.7° produced by the uncompensated circuit.

TABLE A: Second-Order Compensation Phase and Magnitude Behavior

$\dfrac{\omega}{\beta\omega_T}$	PHASE (DEGREE)	MAGNITUDE	20 LOG (MAG) IN dB
0.01	-5.7×10^{-5}	1.0000	0.0000
0.02	-0.0005	1.0004	0.0035
0.04	-0.00367	1.0016	0.0139
0.05	-0.00716	1.0025	0.0217
0.06	-0.01238	1.0036	0.0312
0.07	-0.01965	1.0049	0.0425
0.08	-0.0293	1.0064	0.0554
0.1	-0.0573	1.01	0.0864
0.2	-0.458	1.04	0.3404
0.3	-1.547	1.0896	0.745
0.4	-3.662	1.1576	1.271
0.5	-7.162	1.25	1.938
0.7	-14.428	1.3766	2.776
0.8	-27.1125	1.4598	3.2858
1.0	-45.0	1.4125	3.0

You can model the third-order compensation circuit of Figure 10 in a similar way. Assume that all three op amps in the circuit have a single-pole roll-off in the frequency response represented by:

$$A_O(s) \cong \left(\frac{\omega_T}{s}\right),$$

where ω_T is the amplifier unity-gain crossover frequency.

For ac sinusoidal response, the loop equations can be written:

A_1 loop: $V_O = \dfrac{\omega_T}{s}\left[V_{IN} - \dfrac{1}{\left(1 + K_2\right)}V_2\right]$

A_2 loop: $V_2 = \dfrac{\omega_T}{s}\left[\dfrac{1}{\left(1 + K_1\right)}V_O - \dfrac{1}{\left(1 + K_3\right)}V_3\right]$

A_3 loop: $V_3 = \dfrac{\omega_T}{s}\left[\dfrac{1}{\left(1 + K_2\right)}V_2 - \dfrac{1}{\left(1 + K_3\right)}V_3\right].$

Solving the three simultaneous equations in terms of V_O and V_{IN}, the transfer function is:

$$\frac{V_O}{V_{IN}} = \frac{ax^2 + bx + 1}{ax^3 + bx^2 + \left(1 + \dfrac{1 + K_3}{1 + K_1}\right)x + \left(\dfrac{1}{1 + K_1}\right)},$$

where $a = (1 + K_2)(1 + K_3)$,

$b = 1 + K_2$

$x = \dfrac{s}{\omega_T}$.

Define the time constants $\tau_1 = \dfrac{1 + K_1}{\omega_T}$,

$\tau_2 = \dfrac{1 + K_2}{\omega_T}$, and $\tau_3 = \dfrac{1 + K_3}{\omega_T}$, then substitute,

$$\frac{V_O}{V_{IN}} = (1 + K_1)\left[\frac{1 + \tau_2 s + \tau_2\tau_3 s^2}{1 + (\tau_1 + \tau_3)s + \tau_1\tau_2 s^2 + \tau_1\tau_2\tau_3 s^3}\right].$$

The dc gain of the amplifier is $(1 + K_1)$. And the ac response has the general form:

$$\frac{V_O}{V_{IN}} = (1 + K_1)\varepsilon_p(s),$$

where $\varepsilon_p(s) = \dfrac{1 + \tau_2 s + \tau_2\tau_3 s^2}{1 + (\tau_1 + \tau_3)s + \tau_1\tau_2 s^2 + \tau_1\tau_2\tau_3 s^3}$.

Solve for ac response in terms of τ_1, τ_2, and τ_3:

$$\varepsilon_p(j\omega) = \frac{1 - \tau_2\tau_3\omega^2 + j\tau_2\omega}{1 - \tau_1\tau_2\omega^2 + j\left[(\tau_1 + \tau_3)\omega - \tau_1\tau_2\tau_3\omega^3\right]}.$$

Defining:

$a = 1 - \tau_2\tau_3\omega^2$, $b = \tau_2\omega$,

$c = 1 - \tau_1\tau_2\omega^2$, $d = (\tau_1 + \tau_3)\omega - \tau_1\tau_2\tau_3\omega^3$

$$\varepsilon_p(j\omega) = \frac{a + jb}{c + jd} = \frac{ac + bd + j(bc - ad)}{c^2 + d^2}.$$

The numerator determines the phase shift of the amplifier. Solve the numerator of $\varepsilon_p(j\omega)$:

$$\varepsilon_p(j\omega) = 1 + j\left[(\tau_2 - \tau_1 - \tau_3)\omega - \left(\tau_1\tau_2^2 - \tau_2\tau_3^2 - 2\tau_1\tau_2\tau_3\right)\omega^3\right.$$
$$\left. - \tau_1\tau_2^2\tau_3^2\omega^5\right].$$

To minimize phase shift, make $\tau_2 - \tau_1 - \tau_3 = 0$, and

numerator $\varepsilon_p(j\omega) =$

$$1 + j\left[\left(-\tau_2\right)\left(\tau_1\tau_2 - 2\tau_1\tau_3 - \tau_3^2\right)\omega^3 - \tau_1\tau_2^2\tau_3^2\omega^5\right].$$

The objective is to eliminate the ω^3 term:

$$-\tau_2\left[\tau_1(\tau_2 - 2\tau_3) - \tau_3^2\right] \to 0.$$

First substitute $\tau_1 + \tau_3$ for τ_2, then let $\tau_3 = \alpha\tau_1$, set the equation equal to zero and solve for α:

$$-(\tau_1 + \alpha\tau_1)\left[\tau_1(\tau_1 - \alpha\tau_1) - (\alpha\tau_1)^2\right] = 0$$
$$\alpha^2 + \alpha - 1 = 0.$$

Solving α for the two roots, you obtain:

$\alpha = 0.618034$

$\alpha = -1.618034$

For minimum phase shift, the relationships $\tau_1 + \tau_3 = \tau_2$ and $\tau_3 = 0.618\tau_1$, are used.

$\tau_1 = \tau$

$\tau_2 = 1.618034\tau$

$\tau_3 = 0.618034\tau$,

recalling that $\tau_1 = \dfrac{1 + K_1}{\omega_T}$, $\tau_2 = \dfrac{1 + K_2}{\omega_T}$, and $\tau_3 = \dfrac{1 + K_3}{\omega_T}$.

Since K_1 relates to the dc gain $1/\beta$ of the amplifier as $(1 + K_1) = 1/\beta$, the design equations are:

$$K_1 = \frac{1}{\beta} - 1.$$

Substituting and solving for K_2 and K_3 in terms of K_1,

$K_2 = 1.618034\,K_1 + 0.618034$

$K_3 = 0.618034\,K_1 - 0.382$.

These three last design equations produce optimum phase cancellation for an amplifier with any gain. For example, for an amplifier gain of 10, $K_1 = 9$, $K_2 = 15.18$, and $K_3 = 5.18$.

In order to derive the complete transfer function, the denominator is similarly solved. Therefore, with:

denominator $\varepsilon_p(j\omega) = 1 - 0.618\tau^2\omega^2 - 0.618\tau^4\omega^4 + \tau^6\omega^6$,

the complete ac response is:

$$\varepsilon_p(j\omega) = \frac{1 - j(\tau\omega)^5}{1 - 0.618034(\tau\omega)^2 - 0.618034(\tau\omega)^4 - (\tau\omega)^6}.$$

Finally, the complete amplifier transfer function is:

$$\frac{V_O}{V_{IN}} =$$

$$(1 + K_1)\left[\frac{1 - j(\tau\omega)^5}{1 - 0.618034(\tau\omega)^2 - 0.618034(\tau\omega)^4 - (\tau\omega)^6}\right].$$